1 The monpolistic case with flexible prices

$$\max_p py - TC(y)$$
$$\begin{aligned} s.t. y &= \left(\frac{p}{P}\right)^{-\varepsilon} Y \\ TC &= W\frac{y}{A} \end{aligned}$$

since $y = An$

foc:

$$y + py' - y'MC = 0$$
$$y' = -\frac{\varepsilon}{p}y$$

$$\begin{aligned} y - \frac{\varepsilon y}{p}(p - MC) &= 0 \\ 1 - \varepsilon + \frac{\varepsilon}{p}MC &= 0 \\ p &= \frac{\varepsilon}{\varepsilon - 1}MC \end{aligned}$$

2 Undetermined coefficients

Suppose we have the system

$$\begin{aligned} z_t &= AE_t z_{t+1} + du_t \\ u_t &= \rho u_{t-1} + \varepsilon_t \end{aligned}$$

Notice that in this notation, z_t and u_t may be vectors.

Guess

$$z_t = Cu_t$$

such that

$$\begin{aligned} z_{t+1} &= Cu_{t+1} \\ E_t z_{t+1} &= CE_t u_{t+1} = C\rho u_t \end{aligned}$$

Insert into system to find

$$z_t = (AC\rho + d)\, u_t$$

Compare with guess to find

$$C = AC\rho + d$$

Solve with recursion, that is start with for example $C_0 = I$ and compute

$$C_1 = AC_0\rho + d.$$

Also possible to solve using Kronecker-products, but typically computationally inefficient.

Adding lags:

$$\begin{aligned} z_t &= Bz_{t-1} + AE_t z_{t+1} + du_t \\ u_t &= \rho u_{t-1} + \varepsilon_t \end{aligned}$$

Now, guess that

$$z_t = C_z z_{t-1} + C_u u_t$$

such that

$$E_t z_{t+1} = C_z z_t + C_u \rho u_t$$

Insert into strucutral equation

$$\begin{aligned} z_t &= Bz_{t-1} + A\left(C_z z_t + C_u \rho u_t\right) + du_t \\ \left(I - AC_z\right) z_t &= Bz_{t-1} + \left(AC_u\rho + d\right) u_t \\ z_t &= \left(I - AC_z\right)^{-1} Bz_{t-1} + \left(I - AC_z\right)^{-1}\left(AC_u\rho + d\right) u_t \end{aligned}$$

Compare with guess to find

$$\begin{aligned} C_z &= \left(I - AC_z\right)^{-1} B \\ C_u &= \left(I - AC_z\right)^{-1}\left(AC_u\rho + d\right) \end{aligned}$$

Again, solve with recursion (starting with a guess both for C_u and C_Z