$$
\begin{aligned}
Q_t &= \beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right) \\
Q_t &= \beta E_t \left(C_{t+1}^{-\sigma} C_t^{\sigma} \frac{Z_{t+1}}{Z_t} \frac{1}{\Pi_{t+1}} \right)
\end{aligned}
$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. The tedious way to log-linearize this is to take the differential of the system. On the left-hand side, define $f(C, C_1, Z_1, Z, \Pi) = \beta C_1^{-\sigma} C^{\sigma} \frac{Z_1}{Z} \frac{1}{\Pi}$ =and find

$$
\begin{aligned}
\frac{\partial f}{\partial C} &= \sigma \beta C_1^{-\sigma} C^{\sigma - 1} \frac{Z_1}{Z} \frac{1}{\Pi} \\
\frac{\partial f}{\partial C_1} &= -\sigma \beta C_1^{-\sigma - 1} C^{\sigma} \frac{Z_1}{Z} \frac{1}{\Pi}
\end{aligned}
$$

and so forth. Now we notice that if we multiply each partial with the variable we get back the same result,

$$
\frac{\partial f}{\partial x} x = \sigma \beta C_1^{-\sigma} C^{\sigma} \frac{Z_1}{Z} \frac{1}{\Pi}
$$

which is equal to σQ. Hence, the total differential can be written as

$$
dQ_t = \frac{\partial f}{\partial C} dC_t + \frac{\partial f}{\partial C_1} dC_{t+1} + ...
$$

which we can rewrite as

$$
\begin{aligned}
Q \frac{dQ_t}{Q} &= \frac{\partial f}{\partial C} C \frac{dC_t}{C} + \frac{\partial f}{\partial C_1} C \frac{dC_{t+1}}{C} + ... \\
Q \frac{dQ_t}{Q} &= \sigma Q \frac{dC_t}{C} - \sigma Q \frac{dC_{t+1}}{C} + Q \frac{dZ_{t+1}}{Z} - Q \frac{dZ_t}{Z} - Q \frac{d\Pi_{t+1}}{\Pi} ...
\end{aligned}
$$

The Q drops, and if we set the inflation target to zero such that $\Pi = 1$, and use the magic trick $\frac{dX_t}{X} = \log(X_t) - \log(X) = x_t - x = \hat{x}_t$. We can hence write

$$
\hat{q}_t = \sigma \hat{c}_t - \sigma E_t \hat{c}_{t+1} + E_t \hat{z}_{t+1} - \hat{z}_t - \hat{\pi}_{t+1}.
$$

Finally, $\hat{q}_t = -\hat{I}_t$ and we get

$$
\sigma E_t \hat{c}_{t+1} - \left(\hat{I}_t - \hat{\pi}_{t+1} \right) + \hat{z}_t - E_t \hat{z}_{t+1} = \sigma \hat{c}_t
$$

or, finally,

$$
\hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1} \left(\hat{I}_t - \hat{\pi}_{t+1} \right) + \sigma^{-1} \left(\hat{z}_t - E_t \hat{z}_{t+1} \right)
$$

This is the standard way of writing a log-linearized model. Let us examine some features. First, $\hat{I}_t$ is the log-deviation of the gross nominal interest rate from it's steady state level. That is, $\hat{I}_t = \log(I_t) - \log(I)$. Alternatively, we can write $\hat{I}_t = \log(1 + i_t) - \log(1 + i)$, which is approximately equal to $i_t - i$. Gali sometimes write the relations with some steady state values present, and use log notation in stead of log-deviations. For example, given that in steady state,

$$\frac{1}{I} = \beta C^{-\sigma} C^{\sigma} \frac{Z}{Z} \frac{1}{\Pi}$$

such that $I = \beta^{-1}$. Or, $\frac{1}{1+i} = \frac{1}{1+\rho}$ where $\beta = \frac{1}{1+\rho}$, we get that $i = \rho$. If we insert $\hat{I}_t = \log(I_t) - \log(I) = i_t - \rho$ in the above equation and write out the log-deviations, we note that

$$c_t - c = E_t\left(c_{t+1} - c\right) - \sigma^{-1}\left(i_t - \rho - E_t \pi_{t+1}\right) + \sigma^{-1}\left(z_t - E_t z_{t+1}\right)$$

where we note that $\log(\pi) = 0$ and the same is true since by assumption $Z =$ 1since we assume that the gross inflation target is 1.

Simplifying the above example and using the assumption that $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ such that $E_t z_{t+1} = \rho_z z_t$ gives equation (10) on p. 21 in Galí.