Macro II - Business cycle fluctuations, Lecture 4

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May 3, 2016

Todays Agenda

- Optimal monetary policy design in the NK model - discretion and commitment

Introduction

- Previous discussion:
- No trade-off for central bank. Can achieve optimal allocation
- More realistic with trade-offs. Central bank cannot simultaneously stabilize inflation and the output gap
- Not possible to achieve optimal allocation in general
- Again we push implementation into the background and focus on objectives/criterions for optimal policy

Model

- Welfare loss (proportional to)

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2+\alpha_x\left(x_t\right)^2\right)$$

where $x_t = y_t - y_t^e$, $y^e = y^n$ and $\alpha_x = \frac{\kappa}{\epsilon}$

- Phillips Curve

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa x_t + u_t$$

where, using $\tilde{y}_t = y_t - y_t^n = x_t + \left(y_t^e - y_t^n\right)$, $u_t = \kappa(y_t^e - y_t^n)$ exogenous and follows process $u_t = \rho_u u_{t-1} + \varepsilon_t^u$

- Now a policy tradeoff - pricing depend on $\tilde{y}_t$ but welfare on x_t

Aside: Sources of cost-push shocks

- Anything that yield a short run deviations between flexible price and efficient allocation
- Exogenous shocks to desired price markups - e.g. time varying demand elasticity ϵ_t (Galí App. 5.2)
- Exogenous wage markup shocks (Galí App. 5.2)
- Fluctuations in labor market taxes
- Sticky wages also give rise to a policy tradeoff Erceg, Henderson and Levin (2000) JME

Optimal policy

- Previously: Maximize welfare, i.e., choose sequences $\{i_t\}_{t=0}^{\infty}$ $\{\pi_t\}_{t=0}^{\infty}$ and $\{x_t\}_{t=0}^{\infty}$ to minimize

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2+\alpha_x\left(x_t\right)^2\right)$$

subject to, for all t

$$\pi_t=\beta E_t\left[\pi_{t+1}\right]+\kappa x_t+u_t$$

and Euler equation.

- Note. Suppose central bank alters $\{\pi_t\}_{t=0}^{\infty}$ by changing future inflation. Then the constraint above is affected, leading to different possible choices for π_t and x_t.
- Interpretation: Central bank can promise private sector to follow a designed policy in the future.
- Take into account that policy affects future expectations

Optimal policy under discretion

- First: Other method
- Central bank might have incentive to deviate from announced plan above -not time consistent
- No commitment to future actions.

- Look at payoff today, subject to relevant constraints. Central bank minimizes

$$\pi_t^2 + \alpha_x \left(x_t\right)^2$$

subject to Phillips curve, which we rewrite as, to emphasize the fact that expectations are taken as given,

$$\pi_t = \kappa x_t + v_t$$

where

$$v_t = \beta E_t \left[\pi_{t+1}\right] + u_t$$

- Expectation of tomorrow taken as given
- The first-order conditions is

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t.$$

- Plugging into Phillips curve gives

$$\pi_t = \frac{\alpha_x \beta}{\alpha_x + \kappa^2} E_t \left[\pi_{t+1}\right] + \frac{\alpha_x}{\alpha_x + \kappa^2} u_t$$

- Guess solution $\pi_t = \psi_u u_t$ and solving above equation for ψ_u gives

$$\pi_t = \alpha_x \frac{1}{\kappa^2 + \alpha_x (1 - \beta\rho_u)} u_t \tag{1}$$

and also

$$x_t = -\kappa \frac{1}{\kappa^2 + \alpha_x (1 - \beta\rho_u)} u_t \tag{2}$$

- Note: Solve for optimal policy without using interest rate.

Commitment

- Take into effects of action on future expectations. Minimize welfare loss

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2+\alpha_x\left(x_t\right)^2\right)$$

subject to

$$\pi_t=\beta E_t\left[\pi_{t+1}\right]+\kappa x_t+u_t$$

- The Lagrangian is

$$\mathcal{L}=E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\left(\pi_t^2+\alpha_x\left(x_t\right)^2\right)+\gamma_t\left(\pi_t-\beta\pi_{t+1}-\kappa x_t-u_t\right)\right)$$

  - Law of iterated expectations used to eliminate expectation in $E_t\left[\pi_{t+1}\right]$

- First-order conditions

$$\begin{aligned} \alpha_x x_t - \gamma_t \kappa &= 0 \\ \pi_t + \gamma_t - \gamma_{t-1} &= 0 \end{aligned}$$

  for all t where we set $\gamma_{-1} = 0$ for the case when $t = 0$.
- Can rewrite in terms of inflation an the output gap. For period 0,

$$x_0 = -\frac{\kappa}{\alpha_x}\pi_0$$

  and

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t$$

  for all $t \geq 1$.

- Time inconsistency: consider, say period $t = 3$. Then

$$x_3 = x_2 - \frac{\kappa}{\alpha_x}\pi_3$$

- If the policymaker resolves the problem above in period 3 we instead get

$$x_3 = -\frac{\kappa}{\alpha_x}\pi_3$$

since now $t = 3$ is the initial period.

- Solutions not the same - time inconsistency!

- Can rewrite in terms of
$$x_t = -\frac{\kappa}{\alpha_x}\hat{p}_t$$
where $\hat{p}_t = p_t - p_{-1}$. Proof by induction:
-
$$x_0 = -\frac{\kappa}{\alpha_x}\pi_0 = -\frac{\kappa}{\alpha_x}\left(p_0 - p_{-1}\right) = -\frac{\kappa}{\alpha_x}\hat{p}_0$$
-
$$\begin{aligned} x_1 &= x_0 - \frac{\kappa}{\alpha_x}\pi_1 = -\frac{\kappa}{\alpha_x}\left(p_0 - p_{-1}\right) - \frac{\kappa}{\alpha_x}\left(p_1 - p_0\right) \\ &= -\frac{\kappa}{\alpha_x}\left(p_1 - p_{-1}\right) = -\frac{\kappa}{\alpha_x}\hat{p}_1 \end{aligned}$$
-
$$\begin{aligned} &\vdots \\ x_t &= x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t = -\frac{\kappa}{\alpha_x}\hat{p}_{t-1} - \frac{\kappa}{\alpha_x}\left(p_t - p_{t-1}\right) \\ &= -\frac{\kappa}{\alpha_x}\left(p_{t-1} - p_{-1}\right) - \frac{\kappa}{\alpha_x}\left(p_t - p_{t-1}\right) = -\frac{\kappa}{\alpha_x}\hat{p}_t \end{aligned}$$
- Price level instead of rates of change (inflation), when comparing with discretion.

Solution

- Note that

$$\pi_t = p_t - p_{t-1} = p_t - p_{-1} - (p_{t-1} - p_{-1}) = \hat{p}_t - \hat{p}_{t-1}$$

- Use in NKPC

$$\hat{p}_t - \hat{p}_{t-1} = \beta E_t \left[\hat{p}_{t+1} - \hat{p}_t\right] + \kappa \left(-\frac{\kappa}{\alpha_x} \hat{p}_t \right) + u_t$$

or, where $a = \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$,

$$\hat{p}_t = a\hat{p}_{t-1} + a\beta E_t \left[\hat{p}_{t+1}\right] + au_t$$

- Guessing that solution is of form

$$\hat{p}_t = \psi_p \hat{p}_{t-1} + \psi_u u_t$$

and solving gives, where $\delta = \frac{1-\sqrt{1-4\beta a^2}}{2a\beta}$,

$$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u} u_t$$

- Also,

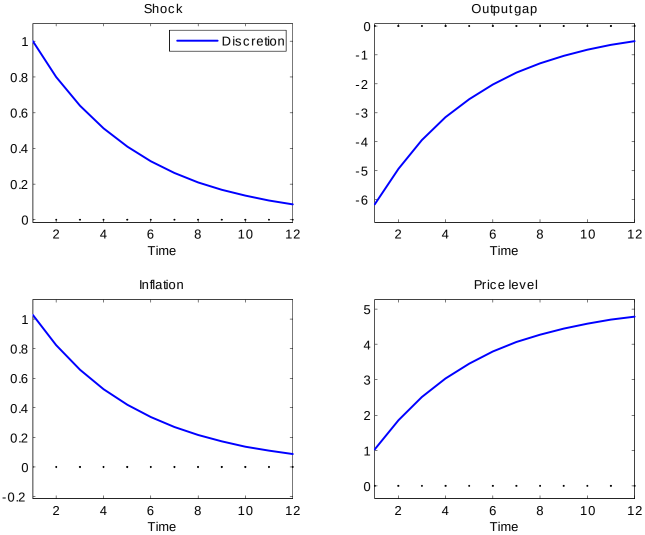
$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x (1-\delta\beta\rho_u)} u_t$$

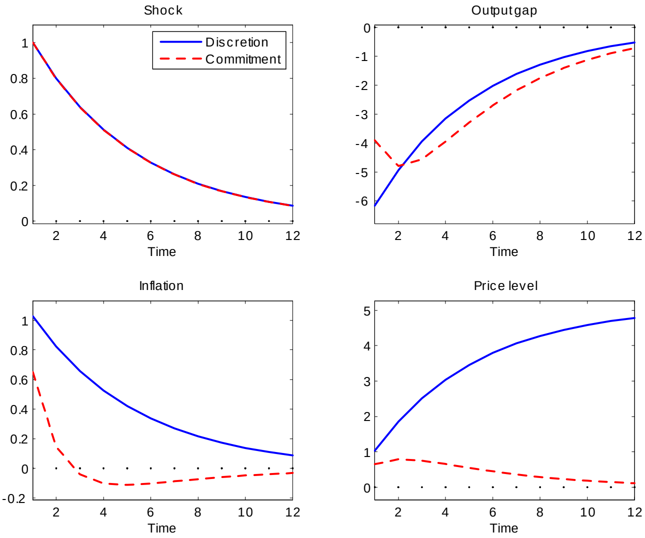
Impulse responses

- One percent shock
- Process

  $$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

  where $\rho_u = 0.8$ (something wrong in figure 5.2 in book where text states $\rho_u = 0.5$)
- Remaining calibration as before.


Shock
Discretion
Outputgap
Inflation
Price level
Time
Time
Time
Time


Shock
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Time

- By affecting expectations the central bank can obtain a better trade-off
- Committing to lower output gaps in the future keeps down inflation today
- Optimal promise - Bring back the price level in the long-run
- Not possible to make such promises under discretion
- Stabilization bias of discretionary policy - failing to internalize short-term benefits from medium-term output-gap deviations.