Macro II - Business cycle fluctuations, Lecture 3

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- Solving rational expectations models general method
- Solving the NK model
- Optimal monetary policy design in the NK model simple rules

$$\begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$
(1)

- The objective is to find "a solution" to this system.
- That is: given initial condition $x_{1,0}$ and $\{\varepsilon_t\}_1^\infty$, what is $\{x_{1,t}\}_1^\infty$ and $\{x_{2,t}\}_1^\infty$?

Example

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A reduced form of the NK model. Consider the NK Phillips curve, but for simplicity assume that the output-gap follows an exogenous process:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$
$$x_t = \rho x_{t-1} + \varepsilon_t$$

$$\begin{bmatrix} x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ -\beta^{-1} \kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

Method of undetermined coefficients (Gali)

• Guess and verify approach, guess that

$$\pi_t = c x_t$$

• Then
$$E_t \pi_{t+1} = c E_t x_{t+1} = c
ho x_t$$

• Insert into Phillips-curve to find

$$\pi_t = (\beta c \rho + \kappa) x_t$$

• Comparing with the guess, we see that $c = \beta c \rho + \kappa$, or that

$$c=rac{\kappa}{1-eta
ho}$$

• This can be generalized to system form, but is inefficient. Instead we look at another much better way of solving.

Solving RE models using the Schur decomposition

Theorem

Given a system of forward difference equations

$$\begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

with initial condition $x_{1,0}$, and where x_1 contains n_1 state variables and x_2 contains n_2 jump variables. Let Z, T be the ordered Schur decomposition of A such that $A = ZTZ^H$. If and only if the number of eigenvalues with modulus smaller than unity equals n_1 , the unique stable solution to the system is given by

$$x_{1,t+1} = Mx_{1,t} + \varepsilon_{t+1}$$

 $x_{2,t} = Cx_{1,t},$

$$\begin{split} M &= Z_{ss} T_{ss} Z_{ss}^{-1}, \ C = Z_{es} Z_{ss}^{-1}, \ T_{ss} = T(1:n_1,1:n_1), \\ Z_{es} &= Z(n_1+1:n,1:n_1), \ Z_{ss} = Z(1:n_1,1:n_1) \end{split}$$

Equilibrium

• New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t \left[\pi_{t+1} \right] + \kappa \tilde{y}_t \tag{2}$$

where

$$\kappa = \lambda \left(\sigma + \varphi \right)$$

Note that, with decreasing returns $(Y_t(i) = A_t(N_t(i))^{1-\alpha})$, the NKPC is qualitiatively identical to above, but with $\kappa = \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ and

$$\lambda = \frac{1-lpha}{1-lpha+lpha\epsilon} \left(1-eta heta
ight) rac{1- heta}{ heta}.$$

Solving forward

$$\pi_t = E_t \sum_{k=0}^{\infty} \beta^k \left(\kappa \tilde{y}_{t+k} \right)$$

• Inflation proportional to the weighted sum of current and expected future output gaps

Vestin (Sveriges Riksbank)

• Euler or (Dynamic) IS equation

$$\tilde{y}_{t} = E_{t} \left[\tilde{y}_{t+1} \right] - \frac{1}{\sigma} \left(i_{t} - E_{t} \left[\pi_{t+1} \right] - r_{t}^{n} \right)$$
(3)

where r_t^n is the natural rate of interest

$$r_t^n = \rho + \sigma E_t \left[y_{t+1}^n - y_t^n \right]$$

• Note that solving the IS equation forward assuming effects of nominal rigidities vanish asymptotically $\lim_{T\to\infty} E_t(\tilde{y}_{t+T}) = 0$ yields

$$\tilde{y}_{t} = -E_{t} \sum_{k=0}^{\infty} \frac{1}{\sigma} \left(i_{t+k} - \pi_{t+k+1} - r_{t+k}^{n} \right) = -E_{t} \sum_{k=0}^{\infty} \frac{1}{\sigma} \left(r_{t+k} - r_{t+k}^{n} \right)$$

- The output gap is proportional to the sum of current and expected deviations between the real interest rate and the natural counter part
- Simple recursive structure
- To close the model the NKPC and IS Equation need to be supplemented with a description of how *i_t* evolves over time. Thus, with sticky prices real variables cannot be determined independently of monetary policy - Monetary policy is non-neutral.

Monetary Policy

Interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \tag{4}$$

• Using (3) and (4) to eliminate the interest rate, and combine with (2) to get

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t [\tilde{y}_{t+1}] \\ E_t [\pi_{t+1}] \end{bmatrix} + B_T (r_t^n - \rho - v_t)$$
(5)

where

$$A_{T} \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta \phi_{\pi} \\ \sigma \kappa & \kappa + \beta \left(\sigma + \phi_{y} \right) \end{bmatrix}$$
$$B_{T} = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and

$$\Omega = \frac{1}{\sigma + \phi_y + \kappa \phi_\pi}$$

Recall first lecture

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} + \frac{1}{\phi_\pi} \hat{r}_t$$

with solution

$$\pi_t = E_t \sum_{k=0}^{\infty} \left(\frac{1}{\phi_{\pi}}\right)^{(k+1)} \hat{r}_{t+k}$$

when $\phi_{\pi} > 1$. Eigenvalue of stochastic differential equation is $\frac{1}{\phi_{\pi}}$

- Conditions for unique solution with sticky prices
- Eigenvalues of A_T less than one (compare with classical model).
- Messy to check See Bullard and Mitra (2002) JME. Result, given that $\phi_{\rm v}$ and ϕ_{π} are nonnegative,

$$\kappa \left(\phi_{\pi} - 1 \right) + \left(1 - \beta \right) \phi_{y} > 0$$

• Note: If $\phi_\pi > 1$ and $\phi_\gamma > 0$ the above expression holds.

Monetary policy shock

• Set
$$r_t^n - \rho = 0 \ (a_t = 0)$$

Assume v_t follows

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

- Calibration as before
- Sticky prices: $\theta = \frac{2}{3}$. Prices last for nine months (also try $\theta = \frac{4}{5}$)

•
$$\phi_y=$$
 0.5/4 and $\phi_\pi=$ 1.5

• Use undetermined coefficient method to solve: Assume that solution is given by

$$egin{array}{rcl} ilde{y}_t &=& \psi_{yv} v_t \ \pi_t &=& \psi_{\pi v} v_t \end{array}$$

• Plug solution into the system (5) to get

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \psi_{yv} v_t \\ \psi_{\pi v} v_t \end{bmatrix} = A_T \begin{bmatrix} E_t \begin{bmatrix} \psi_{yv} v_{t+1} \\ E_t \begin{bmatrix} \psi_{\pi v} v_{t+1} \end{bmatrix} \end{bmatrix} + B_T (-v_t)$$
$$= A_T \begin{bmatrix} \psi_{yv} \\ \psi_{\pi v} \end{bmatrix} \rho_v v_t + B_T (-v_t)$$

Equation system in coefficients. Divide through by v_t to get

$$\begin{bmatrix} \psi_{yv} \\ \psi_{\pi v} \end{bmatrix} = A_T \begin{bmatrix} \psi_{yv} \\ \psi_{\pi v} \end{bmatrix} \rho_v - B_T$$

or

$$(I - A_T \rho_v) \begin{bmatrix} \psi_{yv} \\ \psi_{\pi v} \end{bmatrix} = -B_T \iff \begin{bmatrix} \psi_{yv} \\ \psi_{\pi v} \end{bmatrix} = -(I - A_T \rho_v)^{-1} B_T$$



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- Flexible prices: Inflation and interest rate moves in the same way
- Sticky prices: Inflation and interest rate moves in opposite directions
- Sticky prices: Output and interest rate moves in opposite directions

• From empirical evidence - Employment should initially responds negatively







- Employment responds negatively
- Real wages still problem
- Inflation responds less (also output responds less but change not as big as for inflation)

• Other types of price-setting frictions

- Taylor (1979) AER Cohorts sets prices/wages in overlapping contracts
- Rotemberg (1982) JPE Quadratic cost of price adjustment
- Calvo /Taylor / Rotemberg gives rise to same shaped NKPC See Roberts (1995) JMCB.
- Menu costs Golosov and Lucas (2007) JPE (much older idea) Fix adjustment cost - Firms chose optimally to adjust or not (State-dependent pricing) In menu cost models the degree of monetary non-neutrality varies with the set up - Midrigan (2010) Econometrica Karadi and Reiff (2014).

- Information frictions
 - Lucas (1972) JET
 - Sticky information Mankiw and Reis (2002) QJE
 - Rational Inattention Mackowiak and Wiederholt (2009) AER

- How should monetary policy be designed
- Optimal policy Here objectives/criterions for optimal policy (implementation pushed into the background - See Galí Ch. 3 for a discussion)
- Using simple rules to get close to optimal policy Rules that doesn't require full knowledge of the model and parameters

- In RBC model from Lecture 2: inflation does not matter at all for the real allocation
- Money-in-the-utility-function: downward-sloping function of the interest rate
- Conclusion: set $i_t = 0$ to maximize cash-holdings and let $\pi_t = -r_t$, the Friedman rule!
- With sticky prices and monopolistic competition, this is no longer true...

Efficiency

- Solve the central planners problem, given technology and preferences
- No transaction frictions
- No capital
- Planner solves

$$\max U\left(\mathit{C}_{t},\mathit{N}_{t}\right)$$

subject to

$$C_{t}(i) = A_{t}(N_{t}(i))^{1-\alpha}$$

where

$$N_t = \int_0^1 N_t(i) di$$

$$C_t = \left(\int_0^1 (C_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

• The Lagrangian is

$$L = U\left(\left(\int_{0}^{1} (C_{t}(i))^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \int_{0}^{1} N_{t}(i) di\right)$$
$$-\int_{0}^{1} \lambda_{t}(i) \left(C_{t}(i) - A_{t}(N_{t}(i))^{1-\alpha}\right) di$$

• The first-order conditions are, with respect to $C_t(i)$ and $N_t(i)$,

$$U_{c}\left(C_{t}, N_{t}\right)\left(\int_{0}^{1}\left(C_{t}\left(i\right)\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\left(C_{t}\left(i\right)\right)^{\frac{\varepsilon-1}{\varepsilon}-1}-\lambda_{t}\left(i\right) = 0$$

$$U_n(C_t, N_t) + \lambda_t(i)(1-\alpha)A_t(N_t(i))^{-\alpha} = 0$$

• The first-order conditions for $N_t(i)$ can be written as

$$U_{n}\left(C_{t}, N_{t}\right) N_{t}\left(i\right) = -\lambda_{t}\left(i\right)\left(1-\alpha\right) A_{t}\left(N_{t}\left(i\right)\right)^{1-\alpha}$$

or

$$U_{n}(C_{t}, N_{t})\left(\frac{C_{t}(i)}{A_{t}}\right)^{\frac{1}{1-\alpha}} = -\lambda_{t}(i)(1-\alpha)C_{t}(i)$$

• The first-order conditions for $C_t(i)$ can be written as

$$U_{c}(C_{t}, N_{t}) \left(\int_{0}^{1} (C_{t}(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{1}{\varepsilon-1}\frac{\varepsilon}{\varepsilon}} (C_{t}(i))^{-\frac{1}{\varepsilon}}$$
$$= U_{c}(C_{t}, N_{t}) C_{t}^{\frac{1}{\varepsilon}} (C_{t}(i))^{-\frac{1}{\varepsilon}} = \lambda_{t}(i).$$

• Combining the above conditions gives

$$U_{n}\left(C_{t},N_{t}\right)\left(\frac{C_{t}\left(i\right)}{A_{t}}\right)^{\frac{1}{1-\alpha}}=-U_{c}\left(C_{t},N_{t}\right)C_{t}^{\frac{1}{\varepsilon}}\left(C_{t}\left(i\right)\right)^{-\frac{1}{\varepsilon}}\left(1-\alpha\right)C_{t}\left(i\right)$$
(6)

Since this holds for all $i \in [0, 1]$ the solution for all $C_t(i)$ is the same for all i implying that $\lambda_t(i)$ and $N_t(i)$ is the same over firms and thus

$$N_t = N_t(i)$$
 and $\lambda_t = \lambda_t(i)$. (7)

• The first-order condition with respect to $C_t(i)$ then simplifies to

$$U_c(C_t, N_t) = \lambda_t$$

and hence, using $\frac{\partial L}{\partial N}$,

$$\underbrace{-\frac{U_{n}\left(C_{t},N_{t}\right)}{U_{c}\left(C_{t},N_{t}\right)}}_{MRS_{t}} = (1-\alpha)A_{t}\left(N_{t}\right)^{-\alpha} = MPN_{t}$$
(8)

- Important lessons
 - Marginal rate of substitution between labor and consumption equal to marginal product (8)
 - Labor choices in different firms identical (7)
 - All goods produced in same amount (6)
- All these three violated in the basic New Keynesian model

Monopolistic competition violates (8)

• In model with flexible prices, we get from profit maximization

$$P_t = \mathcal{M}MC_t = \mathcal{M}\frac{W_t}{MPN_t} \tag{9}$$

• Since the labor market is perfectly competitive,

$$\underbrace{-\frac{U_{n,t}}{U_{c,t}}}_{MRS_t} = \frac{W_t}{P_t}$$
(10)

and hence

$$-\frac{U_{n,t}}{U_{c,t}}=\frac{1}{\mathcal{M}}MPN_t$$

- Although we do not have flexible prices, we are close to the flexible price equilibrium - we look at deviations from flexible price equilibrium
- Flexible price allocation inefficient due to markup (we require $-\frac{U_{n,t}}{U_{c,t}} = MPN_t$ for efficiency)

- Inefficiency can be eliminated through subsidy
- Suppose firms only pay (1 τ) times the consumer wage (modifying (9) but not (10)). Profit maximization now gives

$$P_{t} = \mathcal{M} \frac{(1-\tau) W_{t}}{MPN_{t}} \iff -\frac{U_{n,t}}{U_{c,t}} = \frac{1}{\mathcal{M} (1-\tau)} MPN_{t}$$

Setting

$$(1- au)=rac{1}{\mathcal{M}}$$

eliminates distortions!

Realistic?

Distortions associated with nominal rigidities

Markup variations. Average markup over marginal cost in economy

$$\mathcal{M}_{t} = \frac{P_{t}}{(1-\tau) \frac{W_{t}}{MPN_{t}}} = \frac{P_{t}}{\frac{W_{t}}{MPN_{t}}} \mathcal{M} \iff \frac{\mathcal{M}}{\mathcal{M}_{t}} MPN_{t} = \frac{W_{t}}{P_{t}} = -\frac{U_{n,t}}{U_{c,t}}$$

Again (8) might be violated $\left(\frac{\mathcal{M}}{\mathcal{M}_t}\right)$ not always unity).

In addition - Sticky prices leads to

 $P_{t}\left(i\right)\neq P_{t}\left(j\right)$

implying

$$C_{t}\left(i
ight)
eq C_{t}\left(j
ight)$$
 and $N_{t}\left(i
ight)
eq N_{t}\left(j
ight)$

- To find optimal policy, we need to compute welfare with sticky prices
- We use second-order approximation of utility around the efficient allocation (inefficiencies due to nominal rigidities)

• Some tedious algebra (See Galí Appendix 4.1) gives welfare losses in terms of permanent consumption decline from deviating from the efficent allocation

$$-E_0 \sum_{t=0}^{\infty} \frac{U_t - U_t^n}{U_c C} \approx \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \left(\tilde{y}_t \right)^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right)$$
(11)

- Intuition for welfare losses: Firms have different prices ⇒ different labor demand ⇒ price distribution matters for welfare.
- High inflation in period t leads to high relative price in period t.
- First-order effects are eliminated due to efficiency, while second-order effects remain

- Assume that we initially have no relative price distortions; $P_{-1}(i) = P_{-1}$ for all firms $i \in [0, 1]$
- Also assume optimal employment subsidy
- One way to find optimal policy is to do the following:
- Social planner chooses sequences $\{\pi_t\}_{t=0}^{\infty}$, $\{\tilde{y}_t\}_{t=0}^{\infty}$ and $\{i_t\}_{t=0}^{\infty}$ to minimize

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right)\left(\tilde{y}_t\right)^2+\frac{\epsilon}{\lambda}\pi_t^2\right)$$

subject to private sector optimization.

• Private sector behavior can be summarized by

$$\begin{aligned} \tilde{y}_t &= E_t \left[\tilde{y}_{t+1} \right] - \frac{1}{\sigma} \left(i_t - E_t \left[\pi_{t+1} \right] - r_t^n \right) \\ \pi_t &= \beta E_t \left[\pi_{t+1} \right] + \kappa \tilde{y}_t \end{aligned}$$

where

$$r_t^n = \rho + \sigma E_t \left[y_{t+1}^n - y_t^n \right]$$

• Suppose planner chooses, for all t,

$$i_t = r_t^n$$
 and $\pi_t = 0$ and $\tilde{y}_t = 0$

- Satisfies constraints
- First-best achieved !

 Intuition: Planner can choose variables so that marginal costs for firms induce them to set optimal prices equal to previous prices,

$$P_t^* = P_{-1}(i) \Rightarrow \pi_t = 0$$

Can be seen in Phillips curve by noting that $\tilde{y}_t = 0$ and expectation of $E_t [\pi_{t+1}] = 0$ implies that $\pi_t = 0$.

 If there is initial price distortion, the optimal policy converges to the one above - Yun (2005) AER

Note

$$i_t = r_t^n$$

Thus, the equilibrium nominal interest rate (which is also the real rate since $\pi_t = 0$) in the model with sticky prices is equal to the flexible price real interest rate

• Implementation is tricky: $i_t = r_t^n$ is not a good choice as an interest rate rule - multiplicity in equilibria - add endogenous component to the rule to achive an unique equilibrium (Galí 4.3).

- Above: r_tⁿ and ỹ_t depend on flexible price equilibrium, which we do not observe ("Optimal Policy Rules"). Use steady state value of output and real interest rate instead:
- Modified output gap

$$\hat{y}_t = y_t - \bar{y}$$

and

$$\hat{r}^{n}_{t}=-\sigma\psi^{n}_{y\mathsf{a}}\left(1-
ho_{\mathsf{a}}
ight)\mathsf{a}_{t}$$

• Can try to find rules that are close to optimal

• Taylor Rule, where $\hat{y}_t = y_t - \bar{y}$ which is observable

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

or, where we let $v_t = \phi_y \left(y_t^n - ar y
ight)$,

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

• System similar to before (reinterpreted shock)

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \Omega \begin{bmatrix} \sigma & 1 - \beta \theta_\pi \\ \sigma \kappa & \kappa + \beta \left(\sigma + \phi_y \right) \end{bmatrix} E_t \begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \end{bmatrix} + \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix} (\hat{r}_t^n - v_t)$$

- Difference equation. Can be solved.
- Plug solution into welfare loss and evaluate
- Three cases,







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		Taylor	
ϕ_{π}	1.5	1.5	5
ϕ_y^{n}	0	1	0
Welfare Loss	0.08	1.92	0.002