Macro II - Business cycle fluctuations, Lecture 2

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The Basic New Keynesian Model

- Assume imperfect competition
- Reason: Need that prices can be different, and that changes leads to small changes in demand
- Several consumption goods
- Sticky prices
- Flexible wages

Households

- Several consumption goods $C_t(i)$ - continuum with index $i \in [0,1]$.
- Specifically, the preferences represented by the following payoff function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

  - where

$$C_t = \left(\int_0^1 (C_t(i))^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

  - The budget constraint is

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

  - Similar to before. Note that T_t now might include dividends from firms (due to monopolistic competition).
  - Think of two-good case: $C_t = \left(\frac{1}{2} C_{1,t}^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{2} C_{2,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$, standard CES-case...

- No-Ponzi condition

$$\lim_{T \to \infty} E_t \Lambda_{t,T} B_T \geq 0$$

for all t.

- To solve the model, we solve the problem of the consumer in two steps.
- First, we find the consumption and price index by minimizing expendituers to achieve a given level of the aggregate consumption index,

$$\min_{C(i)} \int_0^1 P(i) \, C(i) \, di$$

$$s.t. \left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} = C$$

Proof.

The lagrangian is

- Implies demand (see Appendix 3.1 in Galí for details)

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

where the relevant aggregate price index is

$$P_t \equiv \left[\int P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \quad (1)$$

- Demand curve for individual goods downward sloped. Important for non-trivial price choice of firms.

- Now choose C_t, N_t and B_t as before, using that (given optimal behavior) we can write

$$\int_0^1 P_t(i)\, C_t(i)\, di = P_t C_t$$

- Other first-order conditions as before

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

and

$$Q_t = \beta E_t\left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right]$$

- Loglinearized version, assuming payoff function as before

$$\begin{aligned} w_t - p_t &= \sigma c_t - \varphi n_t \\ c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - \rho\right) \end{aligned}$$

Firms

- Continuum of firms with index $i \in [0, 1]$
- Goods differentiated. Only one firm produces each good i
- Monopolistic competition
- Technology - $\alpha = 0$ for simplicity

$$Y_t(i) = A_t N_t(i)$$

- Calvo model of price stickiness
- Prices reset in each period with probability

$$1 - \theta$$

- Implied price duration

$$\frac{1}{1 - \theta}$$

Price setting and inflation

- Environment implies aggregate price dynamics (log linearized around a zero inflation steady state - See Galí Appendix 3.2)

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{2}$$

- Inflation depends on (i) The share of firms changing price $(1-\theta)$ and (ii) How much the firms that set prices today change their price $\left(p_t^* - p_{t-1}\right)$.
- Next optimal price p_t^*

Price determination

- Firm choose price such that, letting $\Psi_{t+k}\left(Y_{t+k|t}\right)$ denote the cost function

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t}\right)\right)\right]$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

and where

$$Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$$

- Note that $Q_{t,t+k}$ is the period t valuation of (nominal) dividends paid in period $t+k$ for consumers. When maximizing profits, the firm cares about the values dividends yield to consumers (who own the firm).
- Choose P_t^* that maximize the current market value of the firm while the price remains effective.

Price determination

- Firms optimal price (log linearized around a zero inflation steady state - See Galí)

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[mc_{t+k} - mc + p_{t+k} - p_{t-1} \right] \quad (3)$$

or

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[mc_{t+k} + p_{t+k} \right] \quad (4)$$

where

$$\mu = -mc = \log \mathcal{M} = \log \left(\frac{\epsilon}{\epsilon - 1} \right)$$

- Note: In Gali, marginal costs is a function of output, due to decreasing returns, and hence the notation $mc_{t+k|t}$.

- Intuition: Prices a markup (μ) over current and future expected nominal marginal costs ($mc_{t+k} + p_{t+k}$).
- When price are sticky, firms care about future marginal costs because there is a probability that the price set in period t remains in period $t + k$ (prob θ^k)
- Note: the book uses

$$\widehat{mc}_{t+k} = mc_{t+k} - mc$$

  to rewrite the above expression.
- Also note that, in a flexible price equilibrium, we have $\theta = 0$ and hence,

$$p_t^{n*} = \mu + mc_t^n + p_t^n \Rightarrow mc_t^n = -\mu = mc.$$

  since all firms set the same prices ($p_t^{n*} = p_t^n$)

- Note: The optimal price in (4) with sticky prices can be rewritten as, by using first-order condition in period $t+1$

$$p^*_{t+1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left[mc_{t+k+1} - mc + p_{t+k+1}\right] \qquad (5)$$

- Then we can write

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[(mc_{t+k} - mc) + p_{t+k}\right] \quad (6)$$

- $$\begin{aligned} = \quad & (1-\beta\theta)\left[mc_t - mc + p_t\right] \\ & \underbrace{+(1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left[(mc_{t+k} - mc) + p_{t+k}\right]}_{=\beta\theta E_t\left[p_{t+1}^*\right]} \end{aligned}$$

- $$= \beta\theta E_t\left[p_{t+1}^*\right] + (1-\beta\theta)\left[\underbrace{(mc_t - mc)}_{\widehat{mc}_t} + p_t\right]$$

- Todays optimal price: Weighted average of the expectation of tomorrows optimal price and todays marginal cost.

- Using this and (2) gives

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \lambda \underbrace{[mc_t - mc]}_{\widehat{mc}_t}$$

where

$$\lambda \equiv (1 - \beta\theta)\,\frac{1-\theta}{\theta}$$

- Marginal costs high today⇒ inflation high today!
- Future expected prices high⇒firms wants to set prices higher today

Goods market clearing

- We have

$$Y_t\left(i\right) = C_t\left(i\right)$$

for all $i \in [0,1\}$ and all t.

- Defining

$$Y_t = \left(\int_0^1 \left(Y_t\left(i\right)\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

establishes that

$$Y_t = C_t$$

for all t

- Loglinearizing and using in the Euler equation gives

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - \rho\right) \tag{7}$$

Labor market clearing

- Aggregate demand

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)\, di = \int_0^1 \left(\frac{Y_t(i)}{A_t} \right) di \\
&= \left(\frac{Y_t}{A_t} \right) \int_0^1 \left(\left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \right) di
\end{aligned}
$$

- Loglinearizing

$$
n_t = (y_t - a_t) - \epsilon \int_0^1 (p_t(i) - p_t)\, di
$$

The last term is zero up to a first-order approximation around the zero inflation steady state (Galí Appendix 3.3) and hence

$$
y_t = a_t + n_t
$$

Output gap

- Common concept used
- Here: difference between actual output and flexible price output
- Formally, we define the output gap as

$$\tilde{y}_t \equiv y_t - y_t^n$$

- Other definitions: difference between actual output and steady state output

New Keynesian Phillips Curve

- Rewriting price setting decision in terms of "output gap"
- Marginal costs are

$$\begin{aligned} mc_t &= w_t - p_t - mpn_t \\ &= \sigma y_t + \varphi n_t - (y_t - n_t) \\ &= (\sigma + \varphi)\, y_t - (1 + \varphi)\, a_t \end{aligned}$$

Flexible prices: Let y_t^n denote flexible price output.

$$mc = (\sigma + \varphi)\, y_t^n - (1 + \varphi)\, a_t$$

- Can write sticky price marginal costs as

$$\widehat{mc}_t = mc_t - mc = (\sigma + \varphi)(y_t - y_t^n)$$

- When $y_t > y_t^n$ marginal costs are higher than in steady state, since agents demand higher real wages

Equilibrium

- New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t \tag{8}$$

where

$$\kappa = \lambda (\sigma + \varphi)$$

Note that, with decreasing returns ($Y_t(i) = A_t(N_t(i))^{1-\alpha}$), the NKPC is qualitiatively identical to above, but with $\kappa = \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ and $\lambda = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}(1-\beta\theta)\frac{1-\theta}{\theta}$.

- Solving forward

$$\pi_t = E_t \sum_{k=0}^{\infty} \beta^k (\kappa \tilde{y}_{t+k})$$

- Inflation proportional to the weighted sum of current and expected future output gaps

Equilibrium

- Euler or (Dynamic) IS equation

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{9}$$

where r_t^n is the natural rate of interest

$$r_t^n = \rho + \sigma E_t\left[y_{t+1}^n - y_t^n\right]$$

Equilibrium

- Note that solving the IS equation forward assuming effects of nominal rigidities vanish asymptotically $\lim_{T\to\infty} E_t\left(\tilde{y}_{t+T}\right) = 0$ yields

$$\tilde{y}_t = -E_t \sum_{k=0}^{\infty} \frac{1}{\sigma}\left(i_{t+k} - \pi_{t+k+1} - r^n_{t+k}\right) = -E_t \sum_{k=0}^{\infty} \frac{1}{\sigma}\left(r_{t+k} - r^n_{t+k}\right)$$

- The output gap is proportional to the sum of current and expected deviations between the real interest rate and the natural counter part
- Simple recursive structure
- To close the model the NKPC and IS Equation need to be supplemented with a description of how i_t evolves over time. Thus, with sticky prices real variables cannot be determined independently of monetary policy - Monetary policy is non-neutral.