# Macro II - Business cycle fluctuations, Lecture 1

David Vestin

Sveriges Riksbank

May 11, 2020

# Outline: Second part of Macro II

- This part of the class is about Business Cycle Fluctuations and Policy:
  - Business cycle facts. (DV)
  - A simple Real Business Cycle / Classical Monetary model. (DV)
  - Does money have real effects? (DV)
  - Theoretical Models of Business Cycle Fluctuations with sticky prices. (DV)
    - Sticky prices - The basic New Keynesian model
    - Monetary Policy Design in the New Keynesian model
    - Optimal policy - Policy under Discretion and Commitment
  - Extentions and applications (DV)
  - Exercises. (MR 1L)
  - Dynare Lab. (MR 1L)

# Todays Agenda

- Introduction
    - Brief history of business cycle studies
- Empirical Evidence
    - What are the empirical facts we are trying to understand?
- The RBC model (almost the flex-price equilibrium of the New-Keynesian model)
- Log-linearization
- Monetary policy in a classical monetary model

# (Very) Brief history of business cycle research

- Solow growth model -> Solow residual - variations in output beyond what can be explained by labor and capital
- 60-70: Large econometric models, core based on theory + ad hoc dynamics
- 70-80: Lucas critique, Rational expectations revolution
- 80 - present: RBC - microfounded calibrated rational expectations models
- Late 90 - present: New-Keynesian model (RBC + nominal rigidities)

# Stylized Facts: three approaches

1. Remove trend with HP-filter (classic RBC approach)
   - look at variances and correlations, lags
2. Use structural VAR model to
   - identify IRFs to key shocks
3. Use unadjusted data and explicitly model trend (typical in modern New-Keynesian empirical work)
4. Plus: micro-data in the background. Lately, also at the center stage (HANK)

# Stylized Facts: 1, HP-filter

- HP-Filter (Hodrick and Prescott)
  - Let $x_t$ for $t = 1, 2 \dots T$ be the log of a time series which is composed of a trend $(\tau_t)$ and a cyclical component $(c_t)$, i.e.:
    $x_t = \tau_t + c_t$
  - Given $\lambda$, $\tau_t$, solves

$$\min \left[ \sum_{t=1}^{T} (x_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[ (\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right] \qquad (1)$$

  - First term penalizes variations in $c_t$. Second term penalizes variations in the growth rate of $\tau_t$ scaled by a relative penalty parameter $\lambda$ (usually $\lambda = 1600$ in quarterly data)
  - Cyclical component is then the residual $x_t - \tau_t$
  - The HP-Filter is not uncontroversial, but often used. Many other ways to decompose a time series

# Stylized Facts

- Definition: Procyclical (Countercyclical) variable: The variable is high (low) relative to the trend when output his high relative to the trend.
- What do the data say?

# Stylized Facts

| | | US | | International | | Sweden | |
|---|---|---|---|---|---|---|---|
| | | sd/rsd | corr | sd/rsd | corr | sd/rsd | corr |
| Production | Y | 1.66 | 1 | 1.4–1.9 | 1 | 1.7 | 1 |
| Private consumption | C | 0.76 | 0.90 | 0.7–1.8 | 0.4–0.9 | 1.3 | 0.6 |
| Private investment | I | 2.99 | 0.89 | 2.2–3.5 | 0.4–0.9 | 4.1 | 0.6 |
| Inflation rate | $\pi$ | 0.58 | 0.15 | | | 2.0 | -0.2 |
| Employment | N | 0.84 | 0.81 | 0.5–1.3 | 0.4–0.7 | 0.7 | 0.3 |
| Short interest rate | I | 0.89 | 0.38 | | | 0.3 | 0 |
| Real Wage | W/P | 0.39 | 0.16 | 0.5–1.8 | +/- | 0.9 | 0.2 |

sd is standard deviation and rsd is standard deviation relative to standard deviation of GDP. corr is correlation with GDP.

Sources: US: Business fluctuations in US Macroeconomic Time series by Stock & Watson in Handbook of Macroeconomocs vol 1A eds Taylor & Woodford (1999),International: Danthine & Donaldson (EER 1993) and Sweden: Hassler, Lundvik, Persson & Söderlind (1994).

# Stylized Facts

- Components of Demand
  - $Y$ - S.D. $\approx 1.7$ percent of trend output
  - $C$ - Strongly procyclical. Varies less than $Y$ in the U.S. but this is less clear for other countries
  - $I$ - Strongly Procyclical. Varies three times as much as $Y$
- Employment and Productivity
  - $N$ - Strongly Procyclical. Varies less than $Y$
  - $Y/N$ - Labor Productivity is strongly procyclical (as is TFP/Solow Residual). Varies less than $Y$

# Stylized Facts

- Wages and Prices
  - $W/P$ - Mildly Procyclical in the U.S. but with no stable cyclical pattern internationally. Varies less than $Y$
  - $\pi(=\Delta P/P)$ - Phillips curve estimates says that inflation is procyclical

- Financial variables
  - $I$ - Short interest rate is mildly procyclical (or for sweden not at all)
  - $R$ - Real short interest rate is mildly countercyclical in the US and otherwise mildly procyclical
  - Real long rate no clear cyclical pattern

# Stylized Facts

- Moments above from long spans of data.
- Reduced volatility of business cycles since the mid 80s: "The Great Moderation" (see e.g. Stock and Watson, NBER Macroeconomics Annual 2002) - Luck or improved policy? But... financial crisis and Corona...

# Empirical Evidence

## More Solid Evidence Needed

- Evidence is only suggestive: does not take account of the influence of other variables
- Does not take into account the endogeneity between money and output
  - Friedman and Schwartz (1963): $Corr(\Delta M, \Delta Y) > 0$ and argued: $\Delta M \rightarrow \Delta Y$
- But need to isolate how output responds to *exogenous innovations to* $M$
- So called Vector autoregressions is the most widely used tool that have been used for this purpose

- Proposed by Christopher Sims ("Macroeconomics and Reality", 1980, Econometrica)
- Useful way to organize the data and do forecasting
- Model the dependent variables as functions of its own lags and other endogenous variables in the system
- Given some identifying assumptions, a VAR can be used to answer the question: "How does the economy respond to a monetary policy shock?"

- A VAR with $k$ variables in the vector $Y_t$ is written in *structural form* as

$$A_0 Y_t = \alpha + A(L)\, Y_{t-1} + \varepsilon_t \tag{2}$$

where

$$\varepsilon_t \sim N(0, D) \text{ where } D \text{ is diagonal,}$$

and where we normalize the diagonal terms in $A_0$ to unity.

- However, (2) is not estimated. What we estimate on data is the reduced-form

$$\begin{aligned} Y_t &= \beta + B(L)\, Y_{t-1} + u_t, \\ u_t &\sim N(0, V). \end{aligned} \tag{3}$$

- From (2) and (3), it follows that the $k \times 1$ vector with "fundamental" economic shocks, $\varepsilon_t$, are related to the residuals $u_t$ by the following relation:
$$A_0^{-1}\varepsilon_t = u_t, \tag{4}$$
and that $B(L) = A_0^{-1}A(L)$
- If we define $C = A_0^{-1}$, it follows from (3) and (4) that
$$V = \mathrm{E}\left[u_t u_t'\right] = \mathrm{E}\left[C\varepsilon_t\left(C\varepsilon_t\right)'\right] = CDC'. \tag{5}$$
- Let $P$ be the unique lower-triangular Cholesky decomposition of $V$ with positive diagonal elements, so that $PP' = V$
- If the variables in $Y_t$ have been order so that $A_0$ is lower-triangular, then (5) implies that $P = A_0^{-1}D^{1/2}$ and in this case the $i$'th column in $P$ measures the contemporaneous effects of the fundamental shock $i$ in $\varepsilon_t$

- The most recent prominent work that cemented the mainstream view on the monetary transmission mechanism is the seminal work of Christiano, Eichenbaum and Evans (2005, JPE).
- CEE used short-run restrictions on $A_0$ to identify a monetary policy rule $f(\Omega_t)$ and shocks $\varepsilon_{Rt}$ to monetary policy:

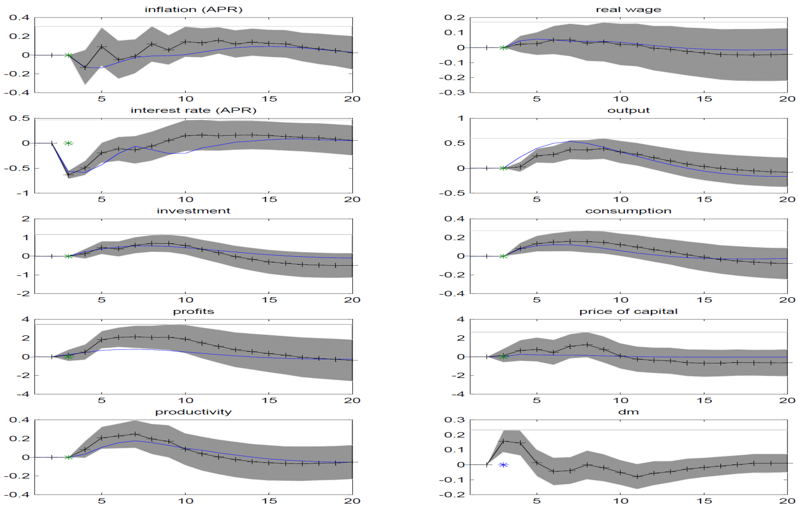
$$R_t = f(\Omega_t) + \varepsilon_{Rt}$$

  where $R_t$ is the FFR and $f$ is a linear of the information set in period $t$, $\Omega_t$.
- CEE partitioned the variables in $Y_t$ as $\left[\begin{array}{ccc} Y_{1t} & R_t & Y_{2t} \end{array}\right]'$ and identified $\varepsilon_{Rt}$ shocks by assuming that
  - Variables in $Y_{1t}$ (GDP, PC, INV, INFL, RW, LP) do not react contemporaneously to $\varepsilon_{Rt}$
  - Variables in $Y_{2t}$ (real profits and money growth) react to $\varepsilon_{Rt}$ but their contemporaneous values do not belong to $\Omega_t$ (i.e. Fed only respond to lags of these variables)

# Empirical Evidence

## CEE: Cemented the Classical View on the Transmission Mechanism

Figure 1: Model and Data Impulse Responses



# Empirical Evidence

## Christiano, Eichenbaum and Evans Monetary Transmission Mechanism

- Main Results:
  1. The interest rate falls for about 1 year
  2. Inflation is slow to react, peaks after 2 years$\Rightarrow$real rates fall
  3. Persistent and hump-shaped effects on real variables
  4. Output, consumption and investment peak after 1,5 years and return to pre-shock levels after about 3 years.
  5. Real profit, real wages and labor productivity rise
  6. Money growth rises immediately
- Walsh: "*We cannot design policy without a theory of how money or monetary policy in general affects the economy*"

# Program: Construct structural economic model consistent with stylized facts

1. Model real side of the economy, understand *efficient* fluctuations (RBC model, Gali chap 2) + nominal flex-price side
2. Add nominal rigidities, examine *inefficient* fluctuations (Gali chap 3)
3. Study how conduct of monetary policy affect business cycle (Gali chap 4+5)
4. Adding bells and whistles: more realistic dynamics

   End product: Christiano, Eichenbaum and Evans + Smets and Wouters
5. Extensions in the aftermath of the financial crisis

# The RBC Model

- RBC idea: Explain business cycle fluctuations in a real model
  - Strong Implications for policy: No (nominal) frictions $\Longrightarrow$ All adjustments (i.e. fluctuations) in real variables are efficient responses to real shocks (technology) - Stabilization policy (including monetary policy) has no role
- Lets see how that works and if it fits the data

# A simple RBC Model (Galí chapter 2)

- Model blocks (Consumers, Production) and Equilibrium
- Study effects of shocks in a classical Real Business cycle model.
- Note: no capital though, typically a standard element of the RBC model (even in the Solow-model)
- Four steps to solve a model
  - Write down optimization problem for agents and take first-order conditions
  - Find steady state
  - Linearize foc around steady state
  - Solve resulting stochastic difference equations using an appropriate method

# Consumers

- Representative household
- Preferences represented by the following payoff function

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

  where $C_t$ is consumption in period $t$, $P_t$ the price level , $N_t$ hours of work, $\beta$ the discount factor.
- Households face the following budget constraint in each period

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

  where $P_t$ is the price of $C_t$, $Q_t$ is the price of riskless bonds $B_t$, $W_t$ is the wage rate and $D_t$ are dividends paid by firms to households (who own the firms)

# Transversality condition

$$\lim_{T \to \infty} \mathrm{E}_t \Lambda_{t,T} B_T \geq 0$$

where

$$\Lambda_{t,T} = \beta^{T-t} \frac{U'(C_T)}{U'(C_t)} \frac{P_t}{P_T}$$

is the (nominal) stochastic discount factor.

- Here we assume additively separable utility

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (6)$$

Note that in the limit when $\sigma \longrightarrow 1$ the first term converges to $\log\left(C_t\right)$.

- To find optimal consumption, hours of work and bond holdings, the consumer then solves the following problem

$$\begin{aligned} & \max_{\{C_T, N_T, B_T\}_{T=t}^{\infty}} E_t \Sigma_{T=t}^{\infty} U\left(C_T, N_T\right) \\ s.t. P_T C_T + Q_T B_T & \leq B_{T-1} + W_T N_T + D_T \end{aligned}$$

subject to the constraints above. The Lagrangian is

$$\begin{aligned} L &= E_t \Sigma_{T=t}^{\infty} U\left(C_T, N_T\right) \\ & -\lambda_T \left[P_T C_T + Q_T B_T - \left(B_{T-1} + W_T N_T + D_T\right)\right] \end{aligned} \tag{7}$$

- The first-order conditions are then

$$\begin{aligned}
\frac{\partial L}{\partial C_t} &= U_{c,t} - \lambda_t P_t = 0 \\
\frac{\partial L}{\partial N_t} &= U_{n,t} + \lambda_t W_t = 0 \\
\frac{\partial L}{\partial B_t} &= -\lambda_t Q_t + \beta E_t \lambda_{t+1} = 0
\end{aligned}$$

- Combining the equations, we we can rewrite this as

$$
\begin{aligned}
-\frac{U_{n,t}}{U_{c,t}} &= \frac{W_t}{P_t} && (8) \\
Q_t &= E_t\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right] && (9)
\end{aligned}
$$

- Using the functional form above,

$$
\begin{aligned}
C_t^{\sigma} N_t^{\varphi} &= \frac{W_t}{P_t} \\
\frac{1}{I_t} &= E_t\left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right]
\end{aligned}
$$

where we have used that $\frac{1}{I_t} \equiv Q_t$, where $I_t$ is the gross interest rate.

# Production

- Technology

$$Y_t = A_t N_t^{1-\alpha}$$

- Note: No capital
- Firms maximize profits, trearing $P_t$ and $W_t$ as given (we assume perfect competition!)

$$\max_{N_t} P_t Y_t - W_t N_t$$

$$s.t. Y_t = A_t N_t^{1-\alpha}$$

FOC:

$$P_t \left(1-\alpha\right) A_t N_t^{-\alpha} - W_t = 0$$

# Model summary

$$\begin{aligned} C_t^{\sigma} N_t^{\varphi} &= \frac{W_t}{P_t} \\ \frac{1}{I_t} &= E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] \\ (1-\alpha)\,A_t N_t^{-\alpha} &= \frac{W_t}{P_t} \\ Y_t &= A_t N_t^{1-a} \\ Y_t &= C_t \end{aligned} \tag{10}$$

$$I_t = I\left(\frac{\frac{P_t}{P_{t-1}}}{\Pi^*}\right)^{\gamma}\exp\varepsilon_t^i$$

- Endogenous variables: $C_t, N_t, Y_t, W_t, P_t, I_t$. 6 variables, 6 equations, looks promising!

# Compressed real model

$$
\begin{aligned}
N_t^{\alpha+\varphi+\sigma(1-\alpha)} &= (1-\alpha)\, A_t^{1-\sigma} \\
Y_t &= A_t N_t^{1-a}
\end{aligned}
$$

$$
N_t = (1-\alpha)^{\frac{1}{\alpha+\varphi+\sigma(1-\alpha)}}\, A_t^{\frac{1-\sigma}{\alpha+\varphi+\sigma(1-\alpha)}}
$$

$$
\log N_t = \frac{\log 1-\alpha}{\alpha+\varphi+\sigma\,(1-\alpha)} + \frac{1-\sigma}{\alpha+\varphi+\sigma\,(1-\alpha)} \log A_t
$$

- $Y_t$ now follows from production function and real wage can be recovered from (10).
- Note: this constitues a solution of the real side of the model, completely independent from monetary policy
- Monetary neutrality

# Log-linearization

- A common method used is to loglinearize the model around a steady state value. We define $x_t = \log X_t$. Then (8) with the payoff function (6) can be written as

$$\sigma c_t + \varphi n_t = w_t - p_t.$$

- For the Euler equation (9) we proceed as follows. Use a first-order Taylor approximation of (9) at the perfect foresight steady state. Rewrite the Euler equation (9) as,

$$1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{1}{Q_t} \right] = E_t \left[ e^{\log \beta - \sigma \Delta c_{t+1} - \pi_{t+1} - q_t} \right] \quad (11)$$

where $\Delta c_{t+1} = c_{t+1} - c_t$ and $\pi_{t+1} = p_{t+1} - p_t$.

# Aside: Taylor approximations

Taylors Theorem: A first-order approximation is

$$f\left(x_1,\ldots,x_n\right) \approx f\left(x_1^*,\ldots,x_n^*\right) + \sum_{i=1}^{k} \underbrace{\frac{\partial f\left(x_1^*,\ldots,x_n^*\right)}{\partial x_i}}_{\text{first-order partials}} \left(x_i - x_i^*\right).$$

- In terms of the model:

$$
\begin{aligned}
f\left(x_1, \dots, x_n\right) &= e^{\log\beta - \sigma\Delta c_{t+1} - \pi_{t+1} - q_t} \\
f(x_1^*, \dots, x_n^*) &= e^{\log\beta - \sigma\gamma - \pi - q}
\end{aligned}
$$

- Also

$$
\frac{\partial f\left(x_1, \dots, x_n\right)}{\partial \Delta c_{t+1}} = -\sigma e^{\log\beta - \sigma\Delta c_{t+1} - \pi_{t+1} - q_t}
$$

and computing the difference$(x_i - x_i^*)$ for consumption,

$$
= \Delta c_{t+1} - \gamma
$$

- A first-order Taylor approximation around a steady state with growth rate $\gamma$ and inflation $\pi$ gives

$$e^{\log\beta-\sigma(c_{t+1}-c_t)+p_{t+1}-p_t-q_t} \approx 1-\sigma\left(\Delta c_{t+1}-\gamma\right)-\left(\pi_{t+1}-\pi\right)-\left(q_t-q\right.$$

- In a perfect foresight steady state, we have

$$1 = e^{\log\beta-\sigma\Delta c-\pi-q} = e^{\log\beta-\sigma\gamma-\pi-q} \iff \log\beta = \sigma\gamma+\pi+q$$

- Hence

$$e^{\log\beta-\sigma(c_{t+1}-c_t)+p_{t+1}-p_t-q_t} \approx 1-\sigma\Delta c_{t+1}-\pi_{t+1}-q_t+\log\beta \quad (12)$$

- Note that $Q_t$ is the price of bonds. In terms of the gross interest rate $I_t$ we can write

$$Q_t = \frac{1}{I_t} \iff Q_t I_t = 1 \iff 1 = e^{q_t+i_t}$$

In steady state, we thus have $q=-i$.

- Using a first-order approximation gives

$$e^{q_t+i_t} \approx e^{q+i} + e^{q+i}(q_t - q) + e^{q+i}(i_t - i) = 1 + q_t + i_t$$

and hence

$$q_t = -i_t.$$

- Then substituting in (12) and using (11) we get

$$1 \approx E_t\left[1 - \sigma\Delta c_{t+1} - \pi_{t+1} + i_t + \log\beta\right]$$

or, solving for $c_t$ and letting $\rho = -\log\beta$ (Household's discount rate)

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right)$$

# Equilibrium

## Fact

*From consumers*

$$\begin{aligned} \sigma c_t + \varphi n_t &= w_t - p_t, \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right). \end{aligned}$$

*Profit maximization*

$$w_t - p_t = a_t - \alpha n_t + \log\left(1 - \alpha\right)$$

## Fact

*(continued) Technology*

$$y_t = a_t + (1 - \alpha)\, n_t$$

*Goods market clearing*

$$y_t = c_t$$

- Labor market clearing

$$\sigma c_t + \varphi n_t = w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

together with goods market clearing and technology gives solution for $y_t$ and $n_t$ as a function of constants and the exogenous disturbance $a_t$ :

$$\begin{pmatrix} \sigma & \varphi + \alpha \\ 1 & -(1-\alpha) \end{pmatrix} \begin{pmatrix} y_t \\ n_t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} a_t + \begin{pmatrix} \log(1-\alpha) \\ 0 \end{pmatrix}$$

## Solution

*We get*

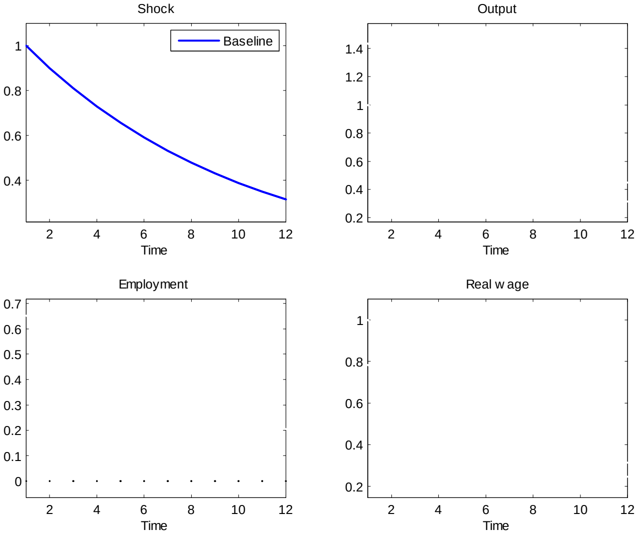
$$
\begin{aligned}
y_t &= \psi_{ya} a_t + \vartheta_y \\
n_t &= \psi_{na} a_t + \vartheta_n
\end{aligned}
$$

*with*

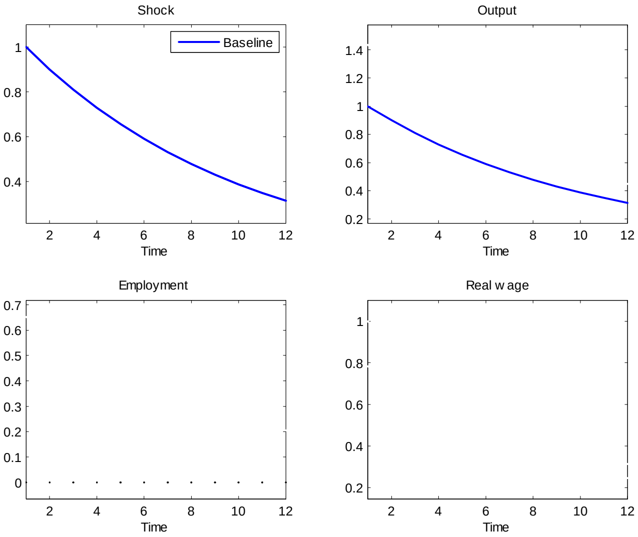
$$
\begin{aligned}
\psi_{ya} &= \frac{1+\varphi}{\sigma+\varphi+\alpha\left(1-\sigma\right)} \text{ and } \vartheta_y = \frac{\left(1-\alpha\right)\log\left(1-\alpha\right)}{\left(1-\alpha\right)} \\
\psi_{na} &= \frac{1-\sigma}{\sigma+\varphi+\alpha\left(1-\sigma\right)} \text{ and } \vartheta_n = \frac{1}{1-\alpha}\vartheta_y
\end{aligned}
$$

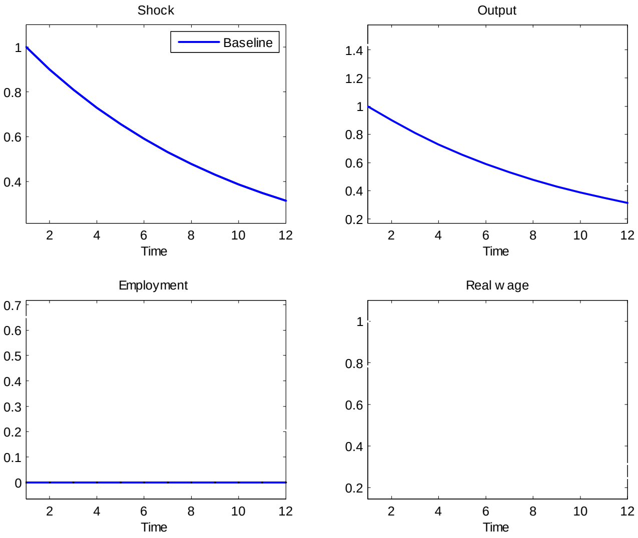
*From this we can solve for all real variables.*

- IMPORTANT: Can solve without money - Money can't drive output!

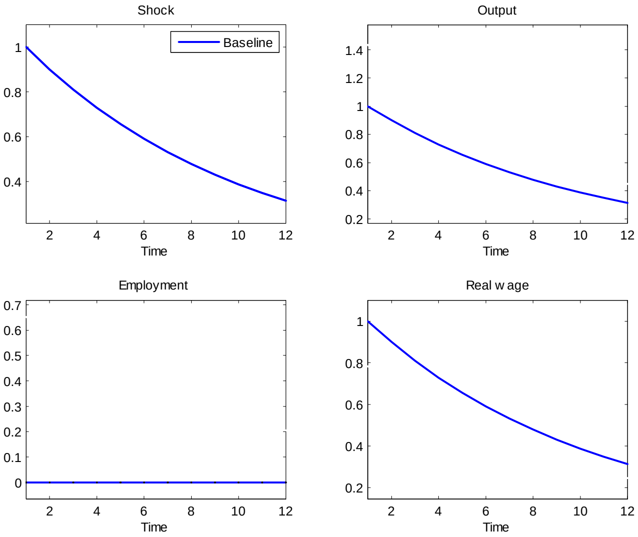


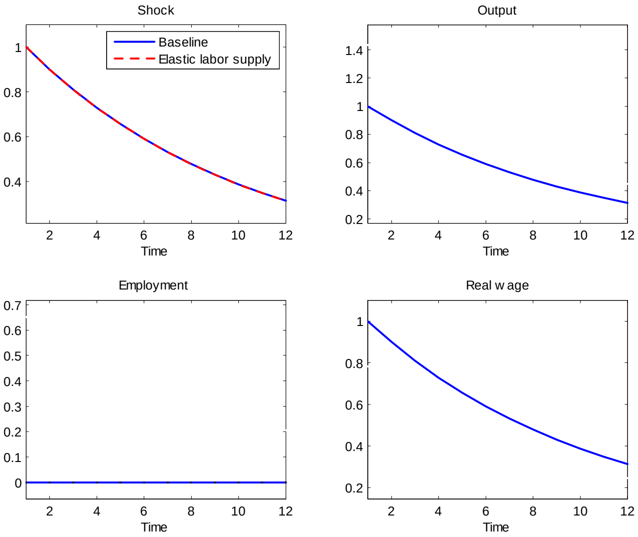
Shock
Baseline
Time
Output
Time
Employment
Time
Real w age
Time

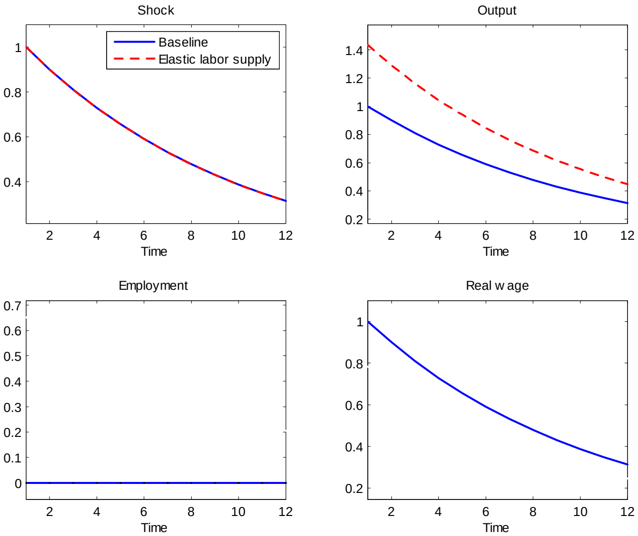

Shock
Baseline
1
0.8
0.6
0.4
2
4
6
8
10
12
Time
Output
1.4
1.2
1
0.8
0.6
0.4
0.2
2
4
6
8
10
12
Time
Employment
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2
4
6
8
10
12
Time
Real w age
1
0.8
0.6
0.4
0.2
2
4
6
8
10
12
Time



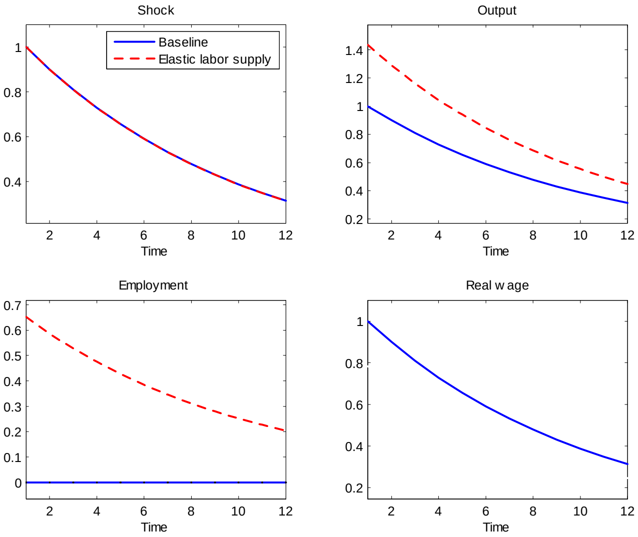
Shock
Baseline
Output
Employment
Real w age
Time

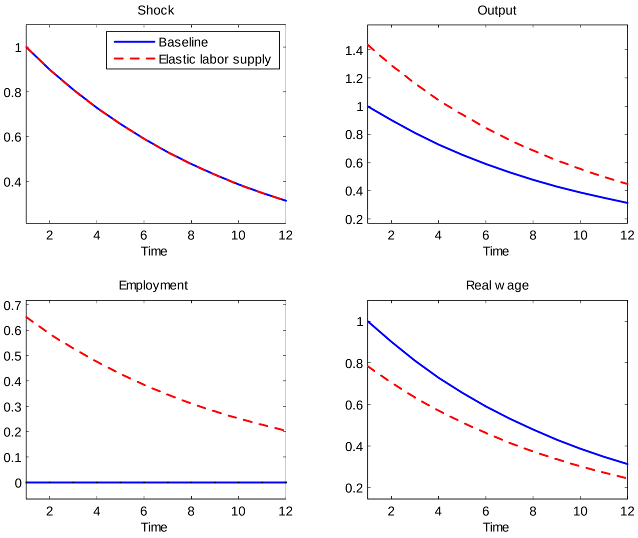

Shock
Baseline
1
0.8
0.6
0.4
2 4 6 8 10 12
Time
Output
1.4
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12
Time
Employment
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2 4 6 8 10 12
Time
Real w age
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12
Time


Shock
Baseline
Elastic labor supply
Output
Employment
Real w age
Time
Time
Time
Time



Shock
Baseline
Elastic labor supply
1
0.8
0.6
0.4
2 4 6 8 10 12
Time
Output
1.4
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12
Time
Employment
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2 4 6 8 10 12
Time
Real w age
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12
Time


Shock
Baseline
Elastic labor supply
Output
Employment
Real w age
Time


Shock
Baseline
Elastic labor supply
Output
Employment
Real w age
Time
Time
Time
Time

- In the first case there is no relationship between ouput and employment - this simple model would thus generate zero correlation betweeen $Y$ and $N$
- Second case better
- Positive correlation between $Y$ and $\frac{W}{P}$
- Both the real wage and employment responds less to the shock - as in the data.
- Very simple model - a larger model would give more reasonable results.

# Monetary policy in the Classical model

- Above only determines relative prices - need monetary policy to determine price level
- Inflation based interest rate rule "Taylor rule"
- Other (not covered here): Exogenous path for interest rate and exogeneous money supply rule.

# Inflation based interest rate rule

- Interest rate determined by

$$i_t = \rho + \phi_\pi \pi_t, \ \phi_\pi \geq 0$$

- Using Fisher identity $r_t \equiv i_t - E_t \pi_{t+1}$ gives

$$r_t - \rho = \phi_\pi \pi_t - E_t \pi_{t+1} \iff E_t \pi_{t+1} = \phi_\pi \pi_t - (r_t - \rho) \quad (13)$$

- Can think of above expression in general terms as (ignoring constants and $r_t$)

$$x_t = a_x x_{t-1} + \varepsilon_t \quad (14)$$

  where we can think of $\varepsilon_t$ as an expectational error. In terms of (13) we have $\varepsilon_t \equiv \pi_{t+1} - E_t \pi_{t+1}$, $\phi_\pi = a_x$ and $(r_t - \rho) = 0$.

# Inflation based interest rate rule (cont)

- We can rewrite (13) as

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} + \frac{1}{\phi_\pi} \underbrace{(r_t - \rho)}_{\hat{r}_t}$$

- Substituting rule in period $t+1$ and so on gives

$$\pi_t = \left(\frac{1}{\phi_\pi}\right)^{T+1} E_t \pi_{t+T+1} + E_t \sum_{k=0}^{T} \left(\frac{1}{\phi_\pi}\right)^{(k+1)} \underbrace{(r_{t+k} - \rho)}_{\hat{r}_{t+k}}$$

- Suppose $\phi_\pi > 1$. Ruling out hyperinflationary equilibria implies that the first term goes to zero as $T \to \infty$.

**Solution**

*We have*

$$\pi_t = E_t \sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{(k+1)} (r_{t+k} - \rho)$$

- This is well defined!
- If $\phi_\pi < 1$ this does not work. Since

$$E_t \pi_{t+1} = \phi_\pi \pi_t - (r_t - \rho)$$

Any process $\pi_t$ such that

$$\pi_{t+1} = \phi_\pi \pi_t - (r_t - \rho) + \xi_{t+1}$$

Now indeterminacy

# Monetary policy shock

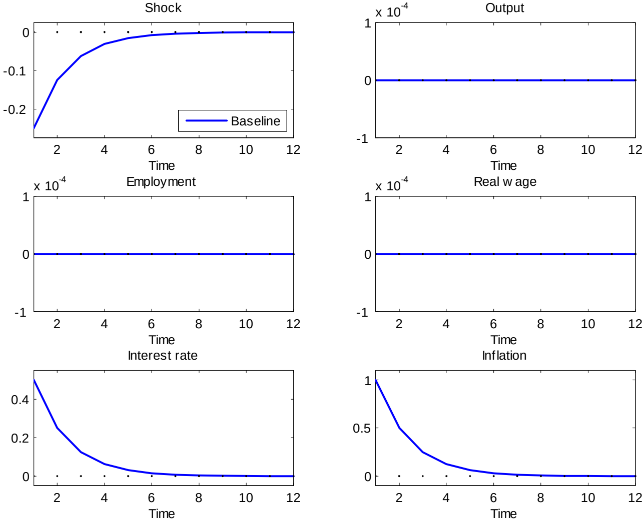
- Add shock to interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

- The shock $v_t$ follows an AR(1) process where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

- Calibration: $\phi_\pi = 1.5$, $\sigma = 1$, $\varphi = 1$, $\alpha = 1/3$ and $\rho_v = 0.5$ with 0.25 percent shock
- Interest rate and inflation moves in the same direction


Shock
0
-0.1
-0.2
Baseline
2 4 6 8 10 12
Time
Output
x 10^-4
1
0
-1
2 4 6 8 10 12
Time
Employment
x 10^-4
1
0
-1
2 4 6 8 10 12
Time
Real w age
x 10^-4
1
0
-1
2 4 6 8 10 12
Time
Interest rate
0.4
0.2
0
2 4 6 8 10 12
Time
Inflation
1
0.5
0
2 4 6 8 10 12
Time

# Exogenous path for interest rate

- Exogenous path for $i_t$ :

$$\{i_t\}_{t=0}^{\infty}$$

  given
- Here: simple case. Just set $i_t =$ constant. Similar to taylor rule above with $\phi_\pi = 0$ - no unique solution.